Overview of Optimization Problems in Electric Car-Sharing System Design and Management

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Abstract

Car-sharing systems are increasingly employing environmentally-friendly electric vehicles. The design and management of Ecar-sharing systems poses several additional challenges with respect to those based on traditional combustion vehicles, mainly related with the limited autonomy allowed by current battery technology. We review the main optimization problems arising in Ecar-sharing systems at strategic, tactical and operational levels, discuss the existing approaches and outline open problems and fruitful research directions.

1 Introduction

The purpose of this paper is to summarize the main contributions to the definition and solution of optimization problems arising in the design and management of car-sharing systems which use electric vehicles.

Car-sharing is a general public mobility mode that is based on the shared use of vehicles by a set of users, who are generally subscribers of the service and pay flat and per-use fees. These systems were introduced around 1970-80 in some limited pilot implementations (see Shaheen et al. [55]), but only recently have seen a considerable development in urban areas. In huge cities congestion and parking costs make the ownership of private cars much less attractive for citizens who rely on public transportation for their regular commuting, and need cars only for special purposes. For a general overview of car-sharing systems we refer to Shaheen et al. [55] and Millard-Ball et al. [42], whereas a recent survey on optimization problems arising in such context was proposed by Jorge and Correia [32]. Finally, the important aspect of demand estimation for car-sharing systems is discussed in Stillwater et al. [56] and Schmöller and Bogenberger [52].

Car-sharing systems are increasingly employing environmentally friendly vehicles that may reduce the overall negative impact of the mobility on the environment, and may have easier access to congested urban areas. The electric vehicles are the most diffused ones among those used by car-sharing systems and in some cases constitute the sole vehicle type available. In this paper, for short we indicate car-sharing systems employing electric vehicles as Ecar-sharing system.

As described in Pelletier et al. [47, 48], several types of electric vehicles actually exist and their characteristics may influence heavily their use possibilities in general and in relation to shared transportation systems. In particular, we consider plug-in electric vehicles (PEVs) that may be charged by plugging-in them to the electric grid. In turn, these vehicles can be classified into plug-in battery electric vehicles (PBEVs), which use the power provided by the battery only, and plug-in hybrid electric vehicles (PHEVs), which can use some recovery energy generated during travel (e.g., from braking) to recharge the battery and can be plugged into the grid, but still also have an internal combustion engine.

For what concerns the organizational issues, an important distinction has to be made between *two-way* (or *roundtrip*) systems, in which the vehicle must be returned to the station where it has been picked up, and *one-way* systems in which vehicles may be also returned to a different station. The second model is clearly more flexible for the users but, as we will extensively discuss in the following, it requires a rebalancing of the vehicles at different stations during the service.

Designing and operating car-sharing systems which use electric vehicles poses additional technological and practical challenges with respect to the systems employing traditional combustion vehicles. For example, the relatively limited autonomy of currently available electric cars requires recharging the vehicles during the day, which has to be performed at specific charging stations. In addition, due to the high costs involved, charging stations are a scarce resource of the territory, and charging times can be quite long unless expensive fast-charge stations are present. Finally, the electricity consumption is considerably affected by the driving and environmental conditions (e.g., the speed profile or the outside temperature) that need to be accurately modeled to better estimate the actual charge status of the vehicles during the day.

In the following sections we examine the main problems that are relevant for the optimal design and management of electric car-sharing systems. For each such problem we both describe the characteristics that have been faced so far in the literature and discuss the components of real-world systems that have not been examined so far, so as to provide interesting and practically motivated research directions.

More precisely, we organized the exposition into two separate sections. The first part (Section 2) is devoted to strategic and tactical problems, which are appropriate in the design of the systems. Within such category falls mainly the problem of locating the charging stations for the electric vehicles and for privately owned cars (Section 2.1). Section 2.2 discusses the tactical problem of defining the allocation strategies for the assignment of vehicles to the stations.

In the second part (Section 3) we present operational problems that arise in the short-term management of Ecar-sharing systems. Section 3.1 introduces the relocation of vehicles between the available stations, which is required to balance the supply and demand patterns. Section 3.2 examines the possibilities offered by battery-swap technologies and Section 3.3 considers the computation of shortest paths specifically designed to incorporate the main characteristics of electric vehicles. Section 3.4 considers the definition of multi-stop travels for electric vehicles that typically occur in freight distribution. Finally, Section 4 draws some conclusions.

2 Strategic and Tactical Problems

As their name suggests, the problems of this class deal with making good high-level, big-picture decisions. These determine the overall structure of the underlying car-sharing system and can therefore have a great impact on how well the system performs. Decisions made at this level are usually long-term, i.e., once they are made, they cannot easily be reversed. Due to their often high cost, they also have a significant impact on the car-sharing operator. Thus, high solution quality is of great importance for these problems. Combined with the fact that strategic decisions need not be made very frequently, this suggests the use of exact or combined methods for solving them.

Although some pilot systems are already in use, not much scientific literature dedicated to the study of the design and operational challenges of Ecar-sharing systems (from a general perspective) exists. Notably, Barth and Todd [6] were among the first to consider the use of electric cars in the context of car-sharing systems. By considering a case study from a resort in Southern California they concluded that (already) 3-6 vehicles are sufficient per 100 trips of each day to satisfy customer waiting times, but approximately 18-24 vehicles would be necessary to also minimize the necessary number of relocations. Besides the number of vehicles per trip, they conclude that the relocation algorithm and the used charging scheme are the main factors for successfully using such a system. Note that particular characteristics of the considered use case include the fact that trips are shorter than 5 miles on average, thus, the charging state of cars never drops below approximately 70%.

Considering a real-world use case from Genoa, Cepolina and Farina [12] are concerned with the design of a flexible, multi-station Ecar-sharing system for pedestrian areas. Their aim is to optimize the dimension and distribution of the fleet among a set of stations at the beginning of operation, so that the sum of total transportation and waiting costs is minimized. Particular characteristics of the system include the possibility for instant access, open ended reservation and one-way trips. A simulated annealing approach that uses a microscopic simulation of user behavior and waiting times is developed, in which a subset of users is assumed to be flexible in the sense that they have an associated set of acceptable stations. Recharging is not explicitly treated but simply assumed to occur in idle times and no explicit relocation actions are considered (i.e., relocation by users). The authors analyze the cost changes with respect to the total number of vehicles and, as in Barth and Todd [6], the influence of the vehicle-to-trip ratio on the total average waiting time.

Other pilot implementations are that of the Kyoto public car system project described in Kitamura [36], and the system with different types of electric vehicles discussed in Luè et al. [40].

Strategic decisions arising in Ecar-sharing systems mainly involve planning locations and sizes (i.e., numbers of charging slots) of charging stations throughout the operational region. The operator's main goal is to minimize their cost arising from building the stations while at the same time ensuring that the profit obtained from satisfied user requests during operation is maximized. Since users will only consider using a car-sharing system if their requests are accepted with a relatively high probability, an operator is facing a difficult trade-off between the initial costs to set up the car-sharing system (long term investment) and the profits obtained later on (operational phase), especially since the latter are highly uncertain.

Tactical decisions are instead related to mid-term planning horizons. Within this time horizon the main optimization problem that is relevant in Ecar-sharing systems is that of allocating the vehicles to the charging stations. Such a problem is mainly relevant for two-way models in which the initial position of the vehicles is critical and may need to be adjusted whenever substantial changes in the demand distribution patterns occur.

2.1 Location of Charging Stations

As mentioned above, a key factor determining the performance of a car-sharing system is the location of each currently unused car within the system, as it determines which customers can actually use it. Since many car-sharing systems are station-based (i.e., cars are always picked up from and returned to a fixed set of parking spots owned by the car-sharing company), the location of these stations becomes equally important. This is especially true for those systems which use electric cars, since they must usually be recharged at the aforementioned stations during the day.

In the following, existing studies on strategic decisions are classified into four categories: (i) location of charging stations in Ecar-sharing systems; (ii) location of charging stations to serve privately owned cars; (iii) location of charging stations for electric taxi cabs; and (iv) location of stations for car-sharing systems with non-electric cars. Note that we include literature related to the latter three categories, as the literature on Ecar-sharing systems is still sparse and as the arising optimization problems share many characteristics. A first brief overview which acts as a guideline to this section's content is given in Table 1.

Table 1: Classification of the literature related with location of charging stations.

Category		Methodology	
type	vehicle type	exact	heuristic / simulation
car-sharing	electric	[8]	
private fleet	electric	[5, 11, 14, 25, 29, 59, 60, 61]	[14, 27, 30, 58]
taxi cabs	electric	[4]	[54]
car-sharing	traditional	[17, 18]	[23]

2.1.1 Location of Charging Stations for Ecar-sharing systems

The only paper considering Ecar-sharing systems is that of Boyacı et al. [8], who describe a bi-objective mixed integer programming (MIP) model for a station-based one-way system. Potential sites for the charging stations are first found by solving a set covering problem. Then the authors seek to optimize the location and size of the stations, together with the number of vehicles, their initial allocation and relocation during the system's operation with respect to both the operator's revenue and the users' benefit. To reduce the size of their model, they use an aggregated model where all relocations happen from or to imaginary hubs, each representing a set of stations, instead of between individual stations. The charge state of each vehicle's battery is not explicitly considered in the model – instead, the necessary pauses for recharging must be provided as an input. The authors evaluate their model for the Nice region by using data from an existing two-way car-sharing system and analyze the effects of various parameters like increased demand on the optimal solution.

2.1.2 Location of Charging Stations for Privately Owned Cars

The most studied case is that of the location of charging stations for privately owned cars. Frade et al. [25] provide an MIP formulation to decide on the location and capacity of electric vehicle charging stations with the objective of maximizing the demand covered under a certain service level and budget constraints. They conduct a case study based on real-world data from Lisbon (Portugal). A similar model is later developed by Cavadas et al. [11] and improved in order to provide a better coverage when some portion of the demand can be transferred between the successive stops of a trip. In addition to transfer of demand, the model is further adapted to a more realistic case where the variation of demand during the day are modeled by splitting the day into time intervals. The comparison of the models using data from Coimbra (Portugal) under different parameter settings reveals two important findings: (i) if there is a possibility of transferring demand, its inclusion in the model might provide significant improvements of the solution; and (ii) independently from its transferability, the consideration of the demand based on different time intervals prevents solutions with overcapacity, which might be the case if demand is aggregated.

Wang and Lin [59] consider a similar objective under budget constraints to decide on the location of multiple types of charging stations which differ in charging speed, and provide an MIP formulation for this problem. They also consider a variant in which the total cost to satisfy all demands is minimized. Both formulations are tested on network from Penghu Island (Taiwan) and the test results show that the consideration of mixed stations yields benefits in terms of objective values compared to using a single station type only.

Minimization of the total cost is adopted also by Baouche et al. [5] when deciding on the optimal locations of the charging stations. Based on a survey on the metropolitan area of Lyon (France), they split the surveyed region into several demand clusters and calculate the energy demand at each of them. The MIP formulation they propose then finds the minimum cost set of potential charging stations that covers all energy demands. The cost takes into account both the construction of the stations and the energy demand for traveling to them. In addition, each station has a fixed type that determines how much charging they can provide. The individual state of vehicles, namely their location or charge state, and the temporal component of demand is only considered in an aggregated way.

A similar approach is used by Chen et al. [14] for the Seattle (Wa, USA) area. Their MIP model determines which charging stations should be opened to minimize the total walking distance required for satisfying all demand. They note that a simple greedy heuristic finds solutions of similar quality, but with a significantly higher maximum walking distance.

González et al. [29] seek to find an optimal charging schedule for private electric vehicles in the Flanders region of Belgium with respect to the cost of electricity used. To estimate the recharging demand, traffic data for conventional vehicles is used. While the locations of charging stations that are opened are not considered in their problem variant (they assume that charging can happen at any time and place), the authors note that in their optimal solution, some zones show a charging demand significantly above the average, which suggests that they are prime candidates for the construction of public charging infrastructure. They also show that over 80% of all current trips could be performed with electric vehicles without requiring any charging outside of the owner's home and note that much of the charging required for the remaining vehicles could be done while the owners are at their workplace.

In contrast to the exact methods used above, Ge et al. [27] employ a genetic algorithm to partition a planning area into zones and assign each of them a charging station of appropriate size, using the required energy expenditure as a quality criterion. Their algorithm is then evaluated on a test instance. Similarly, Hess et al. [30] describe a genetic algorithm for placing charging stations to minimize the total trip distances. They use a traffic simulator, modified to account for electric vehicles, to generate data for the inner city of Vienna, on which they evaluate their algorithm.

Wang et al. [58] describe a heuristic algorithm for finding good locations for charging stations serving private electric vehicles, considering both existing gas stations and entirely new spots as potential sites. Their approach considers a number of objectives including demand coverage, factors relating to the power grid and municipal planning factors (which seek to keep the stations away from places where they might impact other traffic). The algorithm is evaluated on data gathered from the city of Chengdu.

An integrated ILP model that optimizes both the location of charging stations and the routing of electric vehicles is given by Worley et al. [60], with the objective being the minimization of the total cost, which consists of the costs for building stations, charging vehicles and driving. Another ILP based algorithm for finding the optimal charging station locations is presented by Xu et al. [61], who consider customer accessibility (both spatial and temporal), number of charging slots and crime safety as relevant factors.

2.1.3 Location of Stations for Electric Taxi Cabs

Electric taxi cab stations represent a good combination of the two previous categories. Sellmair and Hamacher [54] consider the problem of selecting existing taxi stands as possible locations for charging stations and determining the number of charging points per station. By using simulation techniques, customer trips between taxis stands are generated. The simulation is based on the GPS data collected from five conventional taxis in the city of Munich in Germany. The simulation takes the state of charge into account for deciding whether trips can be accepted or not. An iterative heuristic approach is used to determine the number and location of the charging stations.

Asamer et al. [4] present a study based on operational data of a radio taxi provider in the city of Vienna in Austria. Positioning data of approximately 800 taxis over 12 weeks, one for each calendar month, is used. The authors aim to find locations for a limited number of charging stations dedicated to taxis. Instead of assuming taxi stands as possible locations, regions are considered and the exact locations within the selected areas are identified in a post-optimization phase, where various soft constraints need to be considered. The spatially-distributed charging demand is aggregated, meaning that start and end locations of taxi trips within each region are summed up. Based on this data, a set-covering approach is used to model the location problem with the goal of maximizing the coverage of the aggregated demands. The problem is modeled as a MIP and solved using the IBM CPLEX solver.

2.1.4 Location of Stations for Non-Electric Car-Sharing Systems

As noted in this section's introduction, the problem of finding the optimal locations of vehicle depots in conventional (i.e., non-electric) car-sharing systems is closely related to that of finding the locations of charging stations for electric vehicles, since the factors determining a station's quality are similar (e.g., proximity to areas of high demand). One key difference between these two problems is that models for conventional car-sharing usually do not consider the vehicles' fuel state, since gasoline-powered vehicles can be refilled comparatively quickly.

Correia and Antunes [17] describe MIP formulations that optimize the operator's profit by finding the optimal set of vehicle depots that should be opened, as well as their size and the allocation of vehicles among them. Three different models that maximize the operators' profit are studied, in which (i) the operator has full freedom to decide whether or not to accept a potential trip; (ii) all trips need to be accepted; or (iii) trips may only be rejected by the operator if no vehicle is available at the pick-up station. The authors evaluate their model on input data for the Lisbon area in Portugal, and conclude that the operator's profits decrease significantly when all trip requests must be fulfilled. In another publication, Correia et al. [18] analyze the effects of increased user flexibility on the operator's profit. They develop an MIP formulation that allows users to select one of several potential starting and ending vehicle depots for each trip, with the additional option of providing them with information about the availability of cars or parking spaces at the relevant depots. By applying the model to the Lisbon data set from their previous paper, the authors find that the flexible models improve vehicle usage, but increase walking and total travel times.

In contrast to the aforementioned publications, which deal with finding an optimal solution with respect to some measures of quality, others deal exclusively with the simulation and evaluation of solutions. Fassi et al. [23] evaluate the effects of several growth strategies (like increasing the size of stations and opening new ones) on the activity of stations and members, as well as the members' satisfaction with the service.

2.1.5 Summary, Open Problems and Possible Research Directions

The main objectives in the station location problems for (electric and non-electric) car-sharing systems are to minimize the total cost or maximize the total profit of the car-sharing companies. The characteristics of the location of charging stations for privately owned electric cars can be mainly considered in two categories: problems that aim to minimize total cost while satisfying all demand, and problems that aim to maximize demand coverage under budget constraints. Additionally, objectives pertaining to user satisfaction are sometimes considered. This includes, in addition to the aforementioned demand coverage, objectives like minimizing the walking distance of customers.

The objective of maximizing demand coverage in Ecar-sharing systems seems to be an open problem in the literature and has yet only been addressed in the context of electric taxi cabs [4]. As suggested by [59], multiple types of charging stations can be included in location decisions. Such models could also be extended to consider certain characteristics of the electric grid, like varying charging capacity throughout the day. Improved solutions are obtained when possible transfer of charging demand is considered by Cavadas et al. [11] for the stations dedicated to privately owned electric cars. Adaptation of this idea to the Ecar-sharing systems might be worthwhile to investigate. To better capture aspects related to the particular characteristics of electric cars (i.e., very limited range, long recharging times) integrated models combining strategic and operational aspects seem worth investigating. Specifically, the high degree of uncertainty in terms of energy usage for individual trips suggests further investigations of robust or stochastic problem variants. Furthermore, explicitly capturing the trade-off between naturally arising conflicting objectives (such as long term investment costs, short term profits, relative number of accepted user requests) in terms of bi- or multi-objective problem variants seem worth further studies.

More generally, an aspect that is worth investigating is the study of inter-modal people transportation problems that include (electric) car-sharing systems, i.e., to study the integration of (electric) car-sharing with public transportation and other means of transportation. Besides, considering the likely relatively short distances of many car-sharing trips within cities, a study of the trade-off between vehicle cost and vehicle range seems relevant for the case of electric cars.

Another possible avenue of research would be the development of a flexible pricing scheme that considers the variation of demand throughout the network at different times. This might eventually lead to a system where relocation of vehicles is mostly user-based. It is, however, unclear whether such a system would find acceptance among its potential users.

2.2 Allocation of Vehicles to Existing Stations

Besides relocating vehicles between stations (as described in the next sections), most papers do not seem to explicitly optimize the assignment of vehicles to stations. On the contrary, it is typical that vehicles are considered as origin of a given demand and stations are built and dimensioned to satisfy that demand, see, e.g. [14, 27, 29]. Whenever the actual positions of vehicles throughout a certain planning period (typically a day) are considered in an approach (that, e.g., considers a location-routing problem combining the planning of stations or relocations), an (initial) allocation of vehicles is implicitly optimized by not fixing the (initial) status, see, e.g., aforementioned articles by Correia et al. [18] and Boyacı et al. [8]. On the contrary, other articles (such as Baouche et al. [5]) do not consider these temporal components, but simply design a set of stations (with their capacity) in order to be able to fulfill the demand corresponding to the set of vehicles. Clearly, the latter, which in turn is not so different from other classical assignment problems (p-center, set-covering), is more appropriate for car-sharing systems in which only round trips are allowed and issues such as relocation are not important.

One example of a model that considers the initial allocation of vehicles as a decision variable to be optimized is given by Nakayama et al. [46]. The authors describe a genetic algorithm to optimize, among other factors, the number of vehicles within the car-sharing system and their location at the beginning of each day, given a fixed set of charging stations with a similarly fixed number of parking spots. The algorithm is then evaluated on data from an electric car-sharing operator from Kyoto.

2.2.1 Summary, Open Problems and Possible Research Directions

Since the initial placement and allocation of vehicles to existing stations is rarely considered as an explicit optimization problem but rather assumed to be given, no particular objectives and general constraints have been identified.

An interesting aspect that needs further investigation concerns the integration of vehicle allocation with general location and relocation aspects.

3 Operational Problems

We consider here the optimization problem arising in the operational management of Ecar-sharing systems. Such problems may be grouped into two main classes. The first one is related to the within-day optimal relocation of vehicles while the second considers the possibility of exchanging the battery at charging stations so as to restore vehicle autonomy. We also consider some relevant operational problems that have potential connections with the management of Ecar-sharing systems, namely, the electric vehicle shortest path and vehicle routing problems.

3.1 Relocation of Vehicles for Multiple-Stations Car-Sharing

During the last years, the offer of one-way trip mode has experienced an increased popularity in car-sharing services with fleets of conventional or electric vehicles. One-way car-sharing systems

Table 2: Classification of the literature related with vehicles relocation (UB: user-based relocation strategy, OB: operator-based relocation strategy).

Reference	Strategy	Objective	methodology
[7]	UB	min. relocation costs	Simulation
[16]	UB	max. revenue and max. user's benefit	Simulation
[34, 35]	OB	min. relocation cost and rejected demand	Exact/Heuristic/Simulation
[45]	OB	min. relocation costs	Exact
[37]	OB	min. relocation distance	Simulation
[33]	OB	max. profit	Exact/Simulation
[9]	OB	max. number of relocations served	Exact
[8]	OB	max. revenue and max. user's benefit	Exact

can be *free-floating*, in the absence of fixed parking spots, or *station-based*: in the latter case, reservations may be asked from the users. This section is focused on station-based systems. The one-way option allows for a considerable increase in the number of potential customers interested in shared-use cars. This enhanced flexibility has a strong impact on the vehicle distribution in the service-provider network. Without the imposition of round-trips, an imbalance situation can occur and make the problem of ensuring vehicle availability in under-supplied stations a key issue for the system provider. In order to limit the unserviced trips and restrict economic losses of the car-sharing company, two types of relocation strategies may be implemented. In the first one, called *user-based* (UB) strategy, the relocation is decided by the customer itself, whereas in the second one, called *operator-based* (OB) strategy, relocation decision are made by staff operators at a centralized or distributed level. The main characteristics of the papers here examined are presented in Table 2.

3.1.1 User-Based Strategies

From the system provider point of view, the organization of staff-relocation operations can carry an important economic load and cause operational difficulties. In order to alleviate such burden, Barth et al. [7] introduce two user-based relocation mechanisms called trip joining (or ride-sharing) and trip splitting. Reduced prices are offered to customers willing to accept these modifications of their trip mode. The trip demand data they consider is generated from the University of California-Riverside Campus fleet (UCR IntelliShare) historical database. The system offers trip joining when multiple users want to travel from one low-vehicle-quantity station to a high-vehicle-quantity station, and trip splitting in the opposite situation. Given the demand, a discrete-event time-step simulation model is presented. The simulation allows to calculate the reduction in operator-based relocations thanks to trip joining, trip splitting and the two techniques concurrently. Simulation results show that, in most cases, trip splitting proved to be more effective than trip joining in reducing the staff operators workload. Using these user-based techniques, a 42% reduction in the number of relocations is reported.

Clemente et al. [16] apply information and communication technology to the management of a one-way Ecar-sharing system. Real-time monitoring tools are used in order to propose economic incentives to the users, and help the rebalancing of vehicles in the network stations throughout the day. The authors used a timed Petri Net Framework to model the Ecar-sharing system. The customers response to the proposed trip alternatives modifies the random switches in the Petri Net. The proposed simulation model compares the "as-is" situation (no incentives), with two potential "to-be" strategies. In the "to-be" scenarios, users are encouraged to return cars as soon as possible (offline scenario) or to head to empty stations (online scenario); the latter situation requires the online monitoring of the system. Results on the Ecar-sharing system of Pordenone (Italy) are presented where the online scenario proves to be more profitable for the service provider. The authors conclude that relocation decisions rely on appropriate high-level strategic decisions; when such decisions are not accurately taken (e.g., the station fleet size), the relocation policy is not likely to be effective in solving the congestion problems.

3.1.2 Operator-Based Strategies

Contributions by Kek et al. [34] and Kek et al. [35] are motivated by the development of four shared-use vehicle companies in Singapore. The focus is on a multiple-station company that allows one-way trips; the customer also has the flexibility to modify the previously specified return station en-route. In the first paper, a relocation time-stepping simulation model is proposed and applied on a real set of shared-use vehicle data from commercial operations. Two operator-based relocation techniques are proposed. When service level is the main concern, the vehicle relocation from a neighboring station to an under-supplied station should be performed in shortest time (i.e., travel time to the over-supplied station and relocation duration). The inventory balancing strategy aims instead to relocate vehicles in order to gain an equilibrium in the vehicle distribution in the stations. Cost efficiency is the objective of such technique. The simulation model is validated with real commercial data trips over a typical one-month period. The performance is measured in terms of number of relocations; besides, Kek et al. [34] measures time in which parking slots in a station are either full (FPT) or empty (ZVT). The simulated indicators show fidelity in replicating the trends occurring in the real situation; besides, they provide information on the potential cost savings which could be achieved without impacting the level of service. The authors observe that the individual change of the car-sharing systems parameters has no significant performance impact: this is due to the strong interrelation of operating parameter in such systems.

In Kek et al. [35], the authors present a three-phase optimization-trend-simulation (OTS) decision support system for car-sharing operators to determine a set of near-optimal manpower and operating parameters. A MIP in a time-space network determines the lowest-cost resource allocation and vehicle scheduling, given inputs on station characteristics, vehicle relocation costs and historical customer usage patterns. In the second phase of Trend Filtering, the suggested staff and vehicle activities output from phase one are filtered through several heuristics in order to produce a recommended set of operating parameters. Such output parameters are finally used in the relocation simulator previously described in [34]. The solution approach has been tested on real operational data from Singapore. Results show remarkable improvements in the system performance according to the proposed measure of effectiveness.

Considering the same case study of Kek et al. [34, 35] in Singapore, in Nair and Miller-Hooks [45] the aim is finding a least-cost fleet redistribution plan such that most demand scenarios are satisfied. The probability distribution of users demand is defined by data collected with an Intelligent Transportation System infrastructure which enables monitoring of the trips. A stochastic MIP with joint chance constraints is formulated. The feasible region of the problem is nonconvex. Two solution methods are presented: when demand at stations is correlated, an enumeration procedure based on the concept of p-efficient points is applicable; when the demand at each station is assumed to be independent, a cone-generation solution method is used. Solutions of the proposed case study proved to be robust in simulation studies.

Jorge et al. [33] present two methods for implementing operator-based relocation strategies. The strategic decision of location of stations is taken by adapting the model proposed in Correia and Antunes [17] to the case in which the demand between existing stations is not always satisfied. The first relocation method is based on a novel MIP formulation in a time-space network which aims to maximize the daily profit of the car-sharing system. The second method is a discrete event time-driven simulation for testing two real-time relocation policies. Such strategies consider different frequencies for checking whether a station is a supplier (vehicles in excess) or a demander (vehicles shortage). The two solution approaches were applied, independently and in a combined way, to several realistic scenarios in a case study in Lisbon. The optimized relocation decisions for these networks indicated significant potential profit gain with respect to the case of no relocation actions. The optimal solutions of the mathematical model provide upper bounds on the economic gains that are achievable with relocations since its input data are based on full knowledge of future daily trip demands. Even though trip reservation is necessary in the considered system, the simulation results based on real-time policies are remarkable.

Lee and Park [37] propose an operation planner for relocation staff operations in Ecar-sharing systems. The relocation scheme consists of three steps covering the relocation strategy, the action planning and the staff operation planning, respectively. The demand is estimated by using the extensive Jeju City dataset on actual trips consisting of pick-up and drop-off points collected from a taxi telematics system. Relocation is assumed to be carried out during non-operation hours. The third phase is the main focus of the paper. It implements the relocation staff operations (i.e., moving from an initial to a final station). Single relocation team scheduling is considered for simplicity. The scheduling phase is tackled by using a genetic algorithm in which the relocation distance is the main performance metric considered.

In Bruglieri et al. [9], the authors claim that relocation activities which rely on a truck for auto transport may not be practically implementable in urban environment, since stations may be hardly reachable by the trucks. To overcome this problem, they propose the use of folding bicycles for staff operators relocation movements from an under-supplied station (drop-off) to an over-supplied station (pick-up). Such relocation approach generates a specific pickup and delivery problem called the Electric Vehicle Relocation Problem (EVRP). Given a set of pick-up and drop-off requests defining the network graph, the relocation is formulated as a Vehicle Routing Problem aiming to maximize the total number of requests served. Their MIP model explicitly considers the battery degradation profile using linear assumption. The estimation of the demand has been performed by studying historical data on private car movements in the city of Milan, and restricting these data to the estimated percentage of users interested in using the car-sharing service. A car-sharing simulator has estimated the unbalances due to the projected travel demand. Computational results on realistic instances show that using two workers with a duty time of 5 hours is sufficient to satisfy a high percentage (about 86%) of the relocation requests.

Boyacı et al. [8] present an integrated (strategic, tactical and operational) framework to decide on the location of stations (see Section 2.1.1), on the number of parking slots to satisfy the uncertain user demand, on the assignment of users to slots and on the operator-based relocation actions. The considered Ecar-sharing system is one-way, non-free-floating and reservation-based: both the beginning and the ending station of the trip have to be specified. Demand centers represent sites that can be served by the same set of candidate stations; demands are obtained by an aggregation of orders of rentals, sharing the same set of origin and destination points and common departure and arrival time intervals. The considered graph is a time-space network. A set of scenarios is considered for coping with the stochasticity and seasonality of the demand. The authors develop a bi-objective MIP model. An aggregated model which uses the concept of virtual hubs is presented for the practical solution of instances based on the large-scale car-sharing system in Nice. Extensive sensitivity analysis for relevant parameters is performed. The model evaluates the trade-off between operator benefit and users' level of service, showing that the investment in relocation personnel is worthy both from the company and customers point of view.

3.1.3 Summary, Open Problems and Possible Research Directions

We now summarize the main constraints and optimization objectives considered in the literature for relocation in Ecar-sharing systems.

At each network node, each activity is restricted to begin after the previous one is completed (see [35]). Taking into account relocation action and maintenance activities, the number of available vehicles is updated during the operating day. A limit on the number of rejected demands and vehicle returns is imposed.

There are a number of capacity constraints present in these models. In [35] and [8], station capacity constraints are imposed: in each time discretization step, the sum of available and unavailable vehicles in a station can not exceed the station capacity.
In [35], [45] and [8], the authors limit the number of vehicles relocated out of a station with the number of vehicles available at the start of the planning period; also, the number of vehicles relocated to a station cannot exceed the number of available slots. These conditions are called capacity constraints.

When time-space network representation is used (see [33]), the vehicle flow at each node in the time-space network must be preserved. The stations must have enough parking spaces for vehicles present at each minute. Flow conservation constraints are also considered in [9] and [8]. In [8], atom-coverage constraints are introduced. An atom is a small geographic area that is eligible to receive the car-sharing service. The number of operating parking spaces in all open stations constitutes an upper bound to the number of relocation actions.

In [45], the probabilistic level-of-service constraints state that the redistribution plan must result in inventories that satisfy p-proportion of all demand scenarios in the planning horizon. The resulting system is called a p-reliable system.

In some cases (see [9]) time windows for customers requests are present. Therefore, specific service limitations, such as imposing precedence constraints in the visit time of nodes and bounding the duration of a route are considered.

Finally, specific restrictions characterizing Ecar-sharing systems are imposed in [9] and [8]. In the first paper, the distance traveled by an electric vehicle is assumed to be linearly proportional to the residual charge: it is imposed that an electric vehicle needs to have minimum residual charge (level) in order to perform a trip. In the second paper, the electric vehicles are required to be recharged in the arriving station after each rental operation. In addition, the number of vehicles in the station should be greater than or equal to the number of vehicles requiring charging.

In this specific area there are several open research directions. Regarding the simulation approaches for the impact of user-based relocation strategies, [7] and [16] underline the interest of estimating user participation rate in the proposed relocation activities. The first paper suggests to collect extensive statistical data for making this forecast. The second one proposes a detailed behavioral analysis of the users willingness to accept real time trip suggestions which would permit a more precise trip pricing policy. Other research directions are represented by integrating the relocation action in the strategic planning phase of car-sharing management and to investigate the adoption of real-time relocation policies. In addition, using multiple relocation teams and combining operator-based relocation approach with pricing policies on the parking stations offered to the users, all seem promising options.

Several papers have underlined the strong interrelation between the different levels of decision-making in car-sharing systems problems. As already mentioned, the strategic decision of the location of stations has a huge impact on the tactical and operational issues, such as the routing of the shared-use vehicle fleet, in order to satisfy users requests. An integrated modeling approach seems a promising line of future research.

Car-sharing problems might be considered as a real-world application in which a location-routing scheme is directly present or at least identifiable. The location-routing problem is a research category which considers the integrated solution approaches for tackling location problems in which the tour planning aspects are strongly interrelated with the strategic decisions. To the best of our knowledge, in literature, car-sharing problems have not been explicitly stated in location-routing framework yet and we refer the reader to the survey by Nagy and Salhi [44], which provides a good introduction to the problem. More recently, Prodhon and Prins [50] updates the first survey presenting the multi-echelon problems and several other variants. Finally, the survey by Drexl and Schneider [20] proposes future research directions form the methodological and modeling point of view, such as the integration of revenue management in location-routing formulations.

3.2 Battery Swap

One main challenge for the large-scale spreading of battery-electric vehicles is their limited range and the fact that in contrast to traditional vehicles, re-charging operations take a significant amount of time. Especially for long distance travel, overnight recharging is not sufficient. Thus, battery swapping (rather then recharging) has been considered as a viable alternative, in which the batteries are owned by a company and users simply exchange their currently used (nearly empty) battery with a fully charged one at predefined battery swapping stations (BSSs). A main advantage from a users perspective is that this process can be done in a few minutes (i.e., approximately in the same time frame needed for refueling a traditional car). Even if such technological approach is made difficult by the lack of standardization on batteries and by the huge investments required to set up the system, some interesting studies were presented in the literature.

Yang and Sun [62] study a location-routing problem arising in the delivery of goods to customers using a fleet of electric vehicles (EVs). Given a set of customer demands and of potential BSSs, the goal is to simultaneously determine the location of the battery swapping stations, the allocation of customers to EVs as well as that of EVs to BSSs. In addition, tours from the single depot to serve all customers are designed that consider the selected BSSs and the driving range of the vehicles. The objective is to minimize the total costs arising from the construction of BSSs and the service of the demands with the EVs. Energy consumption and maximum vehicle range are considered to be proportional to the traveled distance. Two flow-based integer programming models are proposed; only the second one allows to revisit BSSs (i.e., to pass at a station / customer multiple times). In addition, two heuristic approaches are studied. The first one is a tabu search which mainly focuses on the location of BSSs and uses a modified Clarke and Wright [15] savings algorithm to heuristically compute a set of routes based on the currently selected swapping stations. A radius-covering method is applied to find an initial set of BSSs. In addition, a hybrid heuristic combining various approaches (namely, modified sweep heuristic, iterated greedy and adaptive large neighborhood search), is described. The main idea is to initially ignore most of the constraints (i.e., battery driving range, BSS location) and subsequently refine a candidate solution to satisfy all conditions. Finally, a last phase aims at improving solutions that are already feasible for the considered problem. Computational experiments are performed using data sets from the CVRP in which all nodes are considered as potential BSSs. Results show that revisits often pay off. The influence of different maximum driving ranges is also analyzed.

Mak et al. [41], who are interested in the location and sizing of BSSs at strategic locations along a network of freeways. They argue that the strategic network decisions need to be taken before observing the actual demand. Therefore, they propose distribution-robust optimization problems where in a first phase the location of BSSs needs to be decided while the number of batteries stored at each BSS can be determined after the uncertain factors are realized. Two variants in which either the expected building and operating costs are minimized ("cost-concerned" model) or a robust estimate of the probability to meet a certain return-on-investment target is maximized ("goal-driven" model) are considered. Models based on mixed-integer second-order cone programming are derived and potential impacts of battery standardization and advancements on the deployment strategy are studied. Computational experiments are performed using instances based on the San Francisco Bay

Area freeway network. It is also pointed out that there exist real world cases (Israel) in which the set of candidate BSSs corresponds to the set of existing gas stations and that upper bounds on the number of batteries per location need to be considered. This restriction arises from the capacity of the electrical grid. Furthermore, the number of arising swap-demanding EVs are treated by a Poisson process, the swapping is assumed to be instantaneous, and a heuristic first-in-first-out strategy for battery selection is considered.

Li [38] studies the scheduling of electric transit buses when either battery swapping or fast charging is employed. An exact branch-and-price algorithm (including stabilization and an initial construction heuristic) as well as heuristic variants based on truncated column generation, variable fixing, and local search are developed. A computational study is performed on instances that are based on publicly available real-world transit data. Besides comparing variants of the proposed algorithms, the results achieved are benchmarked against approaches for other types of buses (gas, diesel, hybrid). Despite the main disadvantage of electric buses, such as the need of deadhead travels to battery stations, the author concludes that the total operational costs of electric buses are smaller than those of the other options. The use of electric buses, therefore, represents a viable alternative also because they produce zero emissions during operation.

Other authors (see, e.g., Chen and Hua [13]) focus on the placement of battery swapping stations without discussing too many aspects that differ from the planning of other re-charging stations; we therefore refer to Section 2.1 for more details.

Another stream of research concerned with battery-swapping deals with the replacement of degraded batteries within a fleet of vehicles by new ones. Almuhtady et al. [1] study different swapping and replacement policies within maintenance of a fleet by a mathematical model as well as two metaheuristic approaches: genetic algorithm and simulated annealing. Experimental results using data inspired from real world are shown.

3.2.1 Summary, Open Problems and Possible Research Directions

Existing approaches in the literature are mainly concerned with either minimizing the total costs in installing (and possibly maintaining) battery-swapping stations. In addition, total routing costs are partially considered in case of classic vehicle routing applications. One exception to this trend is given by Mak et al. [41] who also consider a variant in which the probability to meet a certain return-on-investment goal is maximized. Most of the related works consider constraints limiting maximum travel ranges (whenever a location-routing problem is considered) and restrictions to relatively small sets of potential swapping stations (often only existing “traditional” gas stations). Besides, upper bounds on the numbers of batteries per location arising from limitations of the electric grid are considered (in particular if fast-charging is employed).

Open problems in this area include the appropriate integration of charging times within the overall models and the potential consideration of charging at different speeds instead of assuming a given number of available, charged batteries. Furthermore, integration of aging and replacing aspects of batteries (with respect distance traveled, charging cycles) into battery-swapping problems can be a relevant topic.

3.3 Electric Vehicle Shortest Path Problems

This section discusses optimal path problems involving electric vehicles – with focus on PBEVs – and their specifics. In the car-sharing context these problems might be relevant when the provider wants to estimate the energy consumption of customer trips or when navigation services are offered to customers.

In general one can think of many different practical problem variants of finding an efficient path from A to B while respecting the battery limits (lower and upper bound) of PBEVs. Among them, the following objectives might be relevant:

- minimize energy consumption,
- minimize travel time, and/or
- minimize total costs including costs for traveling, charging, drivers, etc.

Several additional aspects may be considered, e.g.:

- visits to charging stations,
- charging times,
- energy recuperation, i.e., negative energy values on arcs, and/or

- charging station capacities.

An extensive survey on EV shortest path problems and algorithms can be found in Pelletier et al. [47]. In the following, we review important works and extend this survey.

Artmeier et al. [3] minimize energy consumption while allowing recuperation. Since lower and upper bounds of the battery charge have to be respected, the resulting problem is a variant of the constrained shortest path problem which is NP-hard in general. However, here the optimized and constrained resource are the same, finally leading to a polynomial-time algorithm, i.e., a modified Bellman-Ford algorithm. Since the energy consumption on links also depends on the speed on the previous link on the selected path, applying the label-setting algorithm on the original graph is not possible. Thus, the authors describe the construction of an energy graph in which nodes are replicated for each velocity value on incoming arcs. Since the node degree in street network is three on average, the corresponding energy graph is not much larger than the original one.

Eisner et al. [21] extend the work by Artmeier et al. [3] by applying an adaptation of Johnson's potential shifting technique to obtain non-negative edge costs and finally run Dijkstra's algorithm to execute queries in polynomial time. Additionally, the idea of contraction hierarchies is used to further dramatically speed-up shortest path queries.

Sachenbacher et al. [51] also improve the work by Artmeier et al. [3] by considering an A*-related shortest path algorithm. They show that an energy consumption function depending on distance, elevation, and speed provides a consistent heuristic for the A* algorithm, i.e., an energy-optimal route can be found. Their approach significantly outperforms the standard Bellman-Ford and Johnson variants and additionally allows to use dynamic energy information at query-time.

Cassandras et al. [10] consider the problem of finding a path from A to B of a single PBEV with minimal total time while respecting the battery constraints and determining which and how long charging stations are visited. The total time includes both travel and charging times. A non-linear MIP is presented and under several assumptions the authors transform it to an LP: i) at each node there is a charging station with a fixed charging rate, and ii) all energy consumption values on arcs are non-negative. The authors also study the path routing problem with multiple vehicles involving traffic congestion issues and assuming that all vehicles are controlled by a central system. Several non-linear MIPs are proposed to solve this problem.

Arslan et al. [2] deal with an NP-hard minimum-cost path problem for plug-in hybrid electric vehicles (PHEVs) (with both combustion and electric engine) with intermediate fueling/charging stations. They transform the original graph in a way that only origin, destination, and fueling/charging nodes are left. Edges represent the shortest paths between the corresponding nodes in the original graph. When considering only PBEVs, it is possible to find a minimum-cost path from A to B in this graph in polynomial time (e.g., by Dijkstra's algorithm), visiting fueling/charging stations if necessary. For PHEVs, the additional decision of choosing the driving mode makes the problem NP-hard. In an extended problem variant the authors additionally consider vehicle depreciation, stopping, and battery degradation costs. An exact MIP model with quadratic constraints, a dynamic programming and a shortest path based heuristic are presented to solve this problem.

3.3.1 Summary, Open Problems and Possible Research Directions

In earlier works, the main objective is to minimize the energy consumption on the total path. More recently, researchers often consider the minimization of the total travel time while respecting the energy limits, which might be more relevant in practical applications. Additionally, complex cost functions are used combining the (time-dependent) costs for traveling, charging, battery degradation, etc.

The most important common constraints are based on the physical limits of the battery of PBEVs. Because of the currently still quite small battery capacities, PBEVs quickly run out of energy. Recuperation, i.e., the recovery of energy when breaking, may compensate partly for this deficiency. This, however, leads to negative energy values on links and thus to more complicated optimization problems.

The systemic battery limits of PBEVs may also lead to further related constraints: If visits to a given set of charging stations are allowed, then corresponding charging times and station capacities have to be considered, which may also be time-dependent based on the overall state of the underlying electrical grid.

Many authors use simplified formulas to calculate the energy consumption on links. Here, more realistic (possibly non-linear) functions involving a large number of influencing factors may be considered. For some applications, such detailed energy consumption models may not be needed, but nevertheless it should be clear which components mostly contribute to the energy consumption. A sensitivity analysis for a complex energy model might be performed to identify the crucial aspects.

Most works consider only a single vehicle and search for the best path in an egocentric point of view. For governmental stakeholders and local authorities, however, it might to more relevant to consider a global system optimum rather than a local egocentric optimum. Thus, more sophisticated models involving multiple vehicles and complex evaluation functions may be considered in the future.

Realistic energy consumption models and cost functions often involve non-linear terms. Finding accurate linear approximations for these functions might be a way to finally obtain efficient solution approaches for these problems. Discretization might be a promising candidate to reach this goal.

3.4 Electric Vehicles Routing Problem

This section discusses works on vehicle routing problems in which traditional vehicles are either replaced by or mixed with PBEVs or other electric vehicles using alternative fuel. Such problems might be relevant for car-sharing providers if navigation services are offered which involve finding routes visiting a set of locations given by the customer.

Since the battery capacity of electric vehicles is strongly limited, it may be necessary to re-charge the battery along a single route, possibly multiple times. In the literature, this limitation is handled quite differently, as discussed in the next paragraphs. An early survey on sustainable VRP variants can be found in Lin et al. [39]. The survey by Pelletier et al. [47] summarizes several aspects of electric vehicles, i.e., different types of electric vehicles, market penetration, incentives, OR related works, and research perspectives. More details on the specifics of electric vehicles can be found in Pelletier et al. [48]. Since the survey by Pelletier et al. [47] is quite extensive, here we only discuss papers which are particularly relevant or not mentioned in the survey.

In the green VRP introduced by Erdoğan and Miller-Hooks [22], routes for alternative-fuel powered vehicles are determined. A compact MIP based on Miller-Tucker-Zemlin [43] subtour elimination constraints (Big-M) is presented, minimizing the traveled distance while considering the limited distance, possible visits to alternative fuel stations, and upper bounds on the number of tours and their duration. In contrast to classical VRP variants, vehicles are assumed to be uncapacitated here. Refueling time is assumed to be constant, which is usually not the case for electric vehicles. The authors also propose two construction heuristics to create feasible solutions. The results indicate that as the number of fuel stations increases, costs decrease for the same number of served customers, more customers can be served, and the total distance traveled decreases.

van Duin et al. [57] examine the fleet size and mix Vehicle Routing Problem with Time Windows with special focus on different types of electric vehicles for goods distribution. The battery limitations are considered by setting a maximal tour length which can be completed with a single battery charge, i.e., recharging at specific stations is not allowed. A compact MIP based on Big-M constraints is presented without solving the model. To find solutions for a case study in Amsterdam, the authors developed a simple construction heuristic which provides satisfying results in their application.

Schneider et al. [53] extend the green VRP by integrating time windows (VRPTW), customer demands, and capacity constraints to the problem, while focusing exclusively on PBEVs. As a result, recharging times depend on the vehicles battery charge when arrival at a recharging station, and assuming a full recharge. The authors consider a hierarchical objective function first minimizing the fleet size and second minimizing the total travel distance. A hybrid metaheuristic combining variable neighborhood search with tabu search yields small gaps compared to a compact MIP model with Big-M constraints solved by CPLEX.

Frank et al. [26] consider the same problem as Schneider et al. [53], but involve load-dependent energy consumption: each arc is associated with an energy consumption value both for an empty vehicle and a single load unit. Then, the total energy consumption on an arc is linearly dependent on the amount of cargo loaded. The authors provide several linear MIP models for this problem variant: i) a compact model with Big-M constraints, ii) a compact two/three-index-formulation with Big-M constraints allowing at most one charging station visit between two clients, and iii) a set-partitioning model. The same authors present in Preis et al. [49] a more detailed energy consumption model based on distance, altitude, load, and several vehicle properties. In a compact MIP model with Big-M constraints for the electric VRPTW, they minimize the total energy consumption. Additionally, the authors use tabu search heuristics to solve this problem.

Felipe et al. [24] also consider the same problem as Schneider et al. [53] except that i) partial recharges at charging stations are allowed, ii) different charging station technologies can be used at a station (faster charging is more expensive), and iii) the objective is to minimize the charging and battery cycle costs. A compact linear MIP model with Big-M constraints and a simulated annealing approach incorporating local search in several neighborhood structures are proposed.

Goeke and Schneider [28] extend the work by Schneider et al. [53] by considering a mixed fleet with both traditional vehicles and PBEVs in the electric VRPTW. The main contribution of this article is that the energy consumption does not only depend on the distance but involves more parameters, i.e., travel speed, gradient of link, and current load. Here, the energy consumption may also be negative, allowing recuperation and recovery of energy on downward slopes and in breaking events. However, the battery is still fully recharged at a charging station visit. The authors provide a compact MIP model similar to the one in Schneider et al. [53] based on Big-M constraints but including non-linear parts related to load-dependent energy consumption. Additionally, an Adaptive Large Neighborhood Search algorithm is presented. Tests are performed on newly generated instances and on the Solomon-based instances by Schneider et al. [53]. The authors also consider different objective functions not only involving the traveled distance, but also fuel and battery depreciation costs.

Hiermann et al. [31] tackle the same problem as Schneider et al. [53] but additionally consider a mixed fleet of different PBEVs varying in the load and battery capacity. A compact linear MIP model and an adaptive large neighborhood search are presented to solve this variant.

Desaulniers et al. [19] consider a generalization of the classical VRPTW using only electric vehicles: additional nodes represent charging stations which may be visited an arbitrary number of times. The authors also consider several special variants of this problem: i) at most one charging station can be visited on each route, and ii) at each charging station visit the battery is fully loaded. In the more general variant, there is no limit on the number of visited charging stations and the battery may also be partially loaded at a charging station. The results of these variants are compared, leading to the conclusion that in the unrestricted variant routing costs and the number of needed vehicles can be reduced. The authors present exact branch-price-and-cut approaches based on a classical set-partitioning formulation for the considered problem variants. Much effort is put into the development of efficient solution methods for the pricing subproblem, which often represents a performance bottleneck in these approaches. Mono- and bi-directional labeling algorithms are presented for the different variants, enhanced with acceleration strategies based on ng-route relaxations and reduced graphs. To decrease the integrality gap, two sets of valid inequalities defined on the route variables are added: i) the 2-path cuts, and ii) the subset row inequalities. The presented approaches are tested on a benchmark set introduced in Schneider et al. [53] and generated from the classical Solomon VRPTW instances. All instances can be solved in reasonable time. To the best of our knowledge, these approaches represent the computational state-of-the-art for many variants of the electric VRPTW.

Worley et al. [60] consider a combination of location of charging stations and routing of electric vehicles. They present a MIP model with variables for all route segments (no intermediate depot or charging stations) but do not mention how this model with an exponential number of variables is solved. The objective is to minimize the total costs consisting of the costs for building stations, charging vehicles, and driving.

Table 3 gives an overview of the different problem variants discussed in the last two sections.

Table 3: Classification of the literature related with EV routing problems (SP: shortest path problem, VRP: vehicle routing problem).

Reference	Type	Objective	Energy calculation	Charging	Methodology
[3, 21, 51]	SP	min. energy consumption	predefined	no	exact
[10]	SP	min. travel + charging time	predefined	partial	exact
[2]	SP	min. travel + charging costs	distance	full	exact/heuristic
[22]	VRP	min. distance	distance	constant	exact/heuristic
[57]	VRP	min. travel + vehicle + driver costs	distance	no	heuristic
[53]	VRP	min. distance	distance	full	exact/heuristic
[26]	VRP	min. distance	predefined + load	full	exact
[49]	VRP	min. energy	predefined + load	full	exact
[24]	VRP	min. charging + battery costs	distance	partial	exact/heuristic
[28]	VRP	min. distance/battery costs/energy + driver costs	predefined + load	full	heuristic
[31]	VRP	min. travel + vehicle costs	distance	full	exact/heuristic
[19]	VRP	min. distance	predefined	partial/full	exact
[60]	VRP	min. building + charging + travel costs	distance	full	exact

3.4.1 Summary, Open Problems and Possible Research Directions

Most works consider the minimization of the total traveled distance, or more generally the total costs including costs for traveling, fleet investments, battery degradation, etc. Often, the number of vehicles used is minimized in a hierarchical way (in contrast to a weighted objective or a multi-objective formulation). Some authors, however, focus on the minimization of the total energy consumption which seems to be less relevant for practical needs.

Common for many problem variants is the consideration of customer demands, maximal vehicle load capacities, customer time windows, and clearly the highly restricted battery limits. In more strategic problems, the vehicle fleet is heterogeneous in terms of propulsion type (combustion/electric), battery size (if applicable), and/or load capacity.

Similar to Section 3.3, different (more or less detailed) energy consumption models are used. Additionally, for VRP variants it is relevant to also consider the current load for the energy consumption since it may change throughout the tour. The battery limits for PBEVs are considered differently: either simply the tour length is limited or the vehicles are allowed to visit charging stations within the tour. In the second case, different models for charging are implemented: (i) constant charging times, (ii) full charging based on the current state of charge, or (iii) partial charging. Different technologies and therefore charging speeds and capacities may be available at the stations to choose from.

In recent works, the researchers consider more integrated problem variants, e.g., by combining the location of charging stations with the routing part. Here, also the technology, the number of charging points, and the electric capacity may need to be decided for a new charging station.

There are existing models and exact approaches for load-dependent energy consumption. However, there seems to be some room for improvement in terms of model strength and efficiency of solution methods. Also more detailed energy consumption models may be considered in the VRPs, cf. Section 3.3.1.

When considering capacities and technologies of charging stations the corresponding electrical grid and its time-dependent load may be considered. In the area of smart energy grids, researchers brought up the idea of using PBEVs as a temporary energy storage to compensate high demands in peek hours. The integration of such features in existing VRP variants may lead to even more complicated problems but probably would also improve their relevance in real world applications. The combination of the location of charging stations and vehicle routing goes into a similar direction.

4 Conclusions

In this paper, we reviewed the main optimization problems arising in the design and management of car-sharing systems based on electric vehicles. For each problem class, the relevant literature and the main practical issues arising from real-world applications are discussed. Many open problems and possible research directions are discussed, indicating Ecar-sharing systems as a rich an promising research area for optimization methods.

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